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CML Book

The CML book is an online book about how children learn mathematics. Children's Mathematical Learning (CML) was originally designed as a supplement for elementary mathematics education courses including content, methods, and graduate courses. However, parents have also found the materials beneficial. The CML book has descriptions about how children learn math and there are also problems for children to solve. Most problems have data on what percent of children had the problem correct and on how other children solved the problem. The books and the CML project is based on the premise that knowledge of how children learn mathematics will enhance students learning of mathematics-make math more meaningful, and will enhance teachers' ability to teach math to children.

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CML Videos

CML Videos are short video clips of children engaged in mathematical thinking. These videos are designed to illustrate how children learn and think about mathematics. The development of these videos is funded by National Science Foundation grant (DUE 122942). The videos show children in grades K-5 solving mathematical problems. Each video is accompanied by narrative at the beginning, often between clips, and at the end to help viewers focus on how children learn mathematics.

CML videos look at how children learn mathematics in depth. Our videos attempt to show how children think mathematically and how understanding this thinking is a crucial part of teachers' and parent's efforts to further student learning. Our intent is to help viewers realize how several children might solve the same problem. In the third grade video 6 x 8 we attempt to illustrate how children use a variety of methods to solve multiplication problems (See CML video: Multiplication 6 x 8). It is not just an isolated example illustrating the *complexity* of children's mathematical thinking, but rather a series of examples of how children actually learn concepts like multiplication. In CML we typically show several children using different methods solving the same problem. Our intent is for viewers to see the multiple ways that children think. A second key feature is the narration with each video. The narration is designed to focus the viewers' attention on what aspects are important in how children learn mathematics. The narration between clips gives viewers an opportunity to reflect on each child's mathematical thinking. In contrast to other videos, CML videos have a singular purpose of demonstrating how children learn mathematics.

The videos are **free** and available at cmlvideos.com and the password is: cmlvideos.

CML Activities

CML Activities are a collection of math activities we developed over the past 20 years. These are fun math activities designed to help children gain greater mathematical skills in an enjoyable format. Teachers and parents can play the activities with children or children can play with each other. We know that children enjoy doing these activities and that they are learning mathematics. The activities are for sale and are available as a downloadable PDF at CMLactivities.com. These activities are designed to be done with children are intended for readers, teachers and parents, who are working with children.

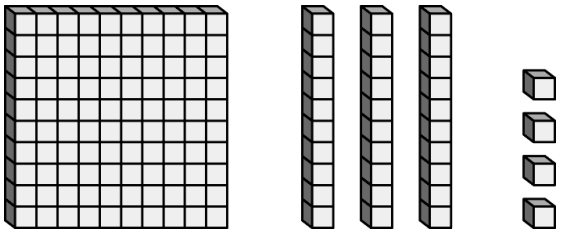
Dedication

This book is dedicated to children, that they might understand mathematics and to future teachers, that they might understand how children understand mathematics.

Introduction

To the student[1]:

Children's Mathematical Learning (CML) materials are designed to be a supplement for a variety of elementary mathematics education courses including content, methods, and graduate courses. These materials have been designed to help you connect **your own learning of mathematics** with **how children understand and learn mathematics**. In addition, the CML Supplement attempts to show how children's learning of mathematics in elementary school is connected to more advanced mathematics in middle and high school. Consequently, this supplement primarily focuses on how children learn and understand mathematics and not on how to teach mathematics. While this supplement is not designed to teach you how to teach mathematics to children, we believe that learning how children think mathematically will be helpful in your teaching of mathematics to children.



Every child is different. Our descriptions of how many or most children learn are not a guarantee of how every child thinks mathematically. The best way to find out what a child is thinking is to ask him/her. The CML Supplement attempts to provide general descriptions of children's learning. These descriptions are intended to help you understand how children approach mathematics differently than adults. By helping you connect children's thinking to your own learning, we hope that this will improve your understanding of both mathematics and children thereby enhance your ability to teach mathematics to children.

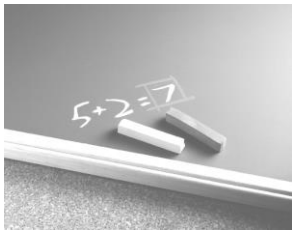
As you use this supplement, try to consider how you learned mathematics, how children learn mathematics, and how you will teach mathematics to children. The more you understand mathematics and how children think about mathematics, the better teacher you will be. Our advice is:

- Read the supplement along with your textbook,
- Do the Problems and Exercises, and
- Try to understand how children learn mathematics.

Organization

[1] Note: Throughout the supplement, **student** refers to you, a preservice elementary teacher or graduate student, and **child** refers to an elementary school student.

Each section contains insights and examples of how children come to understand mathematics. Many of the descriptions of how children learn are based on research, but some are based on the authors' personal experiences with helping children learn mathematics. We have attempted to use our own knowledge and experiences along with research to present descriptions of how children understand the mathematics. We have also attempted to describe the "why" behind the mathematics that you will be teaching to children because, as a teacher, you should be able to explain the mathematics to children. Key underlying, mathematical concepts are presented and discussed. Manipulatives are discussed in relationship to the mathematics children are learning. As a teacher, it is important to understand the mathematics that a manipulative represents or embodies. Occasionally, a few class activities are included to help you see how children think mathematically. Connections are made to children's later developments in mathematics, which might include the mathematics that children will learn in later grades or even middle or high school. In summary, the CML Supplement attempts to connect the mathematics you are learning with the mathematics you will be teaching children.



At the end of each section are "**Problems and Exercises**" for you to solve. These problems and exercises are different than the problems in your mathematics textbook in that they are specifically designed to help you learn how children learn mathematics. Many of these questions are from the National Assessment of Educational Progress (NAEP) and the Third International Mathematics and Science Study (TIMSS). The NAEP tests are only given in the United States and are used to measure national achievement. The TIMSS tests are given in the United States and internationally and used primarily to compare mathematics achievement in the United States with other countries. Both these tests are normally given every two to four years and are used to measure the educational achievement in mathematics in grades 4, 8, and 12. The test questions along with the performance data of children on the test questions are available at the following websites:

NAEP: http://nces.ed.gov/nationsreportcard/itmrls/
TIMSS: http://timss.bc.edu/timss2003i/released.html
http://timss.bc.edu/timss1999i/timss_test.html
http://timss.bc.edu/timss1995i/Items.html

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NAEP
NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS

TIMSS
UNITED STATES

Another key feature of this supplement is "**Children's Solutions and Discussion of Problems and Exercises**" section. Here we present both children's solutions and errors or a discussion for some of the "**Problems and Exercises**". With the NAEP and TIMSS questions, we give the percentage of children who had the correct solution. With some NAEP questions we also give children's actual solutions. All the data we present may not be indicative of how a particular child or class will perform on the problem. The data is intended to provide a general idea on how many children perform on similar problems.

Before looking at the "**Children's Solutions and Discussion of Problems and Exercises,**" be sure to do the problems first and then see how children attempted them. Especially consider what mathematics you will need to understand in order to understand and facilitate children's growth in learning mathematics. To help you reflect on these connections and understandings, each section also has "**Questions for Discussion**". These are general questions for you to discuss, write about, or reflect upon.

Connections to video clips are interspersed throughout the book. The video icon is an indication that a CML video is related to the topic being discussed. Short questions to consider about the video are also included. The videos are **free** and available at cmlvideos.com and the password is: cmlvideos. We encourage you to take some time to watch each video when you see the video icon.

We sincerely hope that you enjoy using CML and find it useful in your learning of mathematics. More importantly, we hope that CML allows you to make important connections to the ways that children learn and think about mathematics.

Preface to the Instructor

Children's Mathematical Learning (CML) is designed to be used with any course and textbook for the mathematical content or methods courses for elementary teachers. We have developed alignment tables that show how CML is aligned with the most widely used textbooks in mathematics education. The tables for textbooks published by Pearson can be found in the CML Instructors Guide and all available tables can be found on our website:
http://www.childrensmathematicallearning.com

Purpose of CML

CML is not specifically designed to teach mathematics, but to help preservice teachers connect the mathematics they are learning with how children learn and understand mathematics. The intent is that this will enhance preservice teachers' understanding of mathematics and consequently improve their future teaching of mathematics to children. Students are more motivated to learn when they can see the connection between their learning and their future profession.

Using CML

We have two main suggestions in effectively using CML. First, having students read CML is essential; consider assigning the reading of the CML section along with the corresponding section that you are covering in the mathematics content textbook. You may assign some or all of the **Problems and Exercises** at the end of each section. These problems are designed for children; some are eighth grade problems so a few may be a bit more challenging. Most answers are in the back of the book. Encourage students to solve these problems first. However, the key is the **Children's Solutions and Discussion of Problems and Exercises.** So, the second main suggestion is to, whenever possible, discuss or cover the data and children's solutions from this section with your students. The data and solutions are designed to provide insights into how children understand mathematics. The **Questions for Discussion** may be more useful in a methods or graduate course in mathematics education.

These are a few suggestions for using CML. We would be interested in learning how others have used our book, especially in methods and graduate mathematics education courses. You can email your comments and suggestions to dfeikes@pnc.edu.

CML and Students

Most students in mathematical content courses for elementary teachers are apprehensive about learning mathematics. Our hope is that preservice teachers not only learn mathematics, but that they begin to develop an understanding of how children think about mathematics, learn to appreciate mathematics, and enjoy teaching it.

With CML, students should never say, "Where will I ever use this?"

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